



The CP properties of the 2HDM

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The 2HDM potential



$$\begin{aligned} V(\Phi_1, \Phi_2) = & -\frac{1}{2} \left\{ m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \right\} \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] \\ & + \left\{ \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right\} \end{aligned}$$

EWSB



- Vacuum expectation values – most general form that preserves $U(1)_{em}$:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}$$



Change of basis

- Initial expression of potential is defined with respect to fields Φ_1 and Φ_2 - this defines our initial basis.
- We can change to a new basis by the following transformation

$$\bar{\Phi}_i = U_{ij} \Phi_j$$

where U is a $U(2)$ matrix.



CP-properties

- Higgs-sector is CP-conserving if a basis exists in which all the parameters of the potential AND the vevs are simultaneously real.
- If CP is broken, it can be broken either *explicitly* or *spontaneously*.
 - Explicit breaking: No basis exists in which the parameters of the potential are all real.
 - Spontaneous breaking: A basis exists in which all the parameters of the potential are real, but the vevs cannot be made real simultaneously.



Motivation

- Will it be possible in experiments to measure if CP is broken *explicitly* or *spontaneously*?
- Will the type of CP-violation (explicit vs spontaneous) have any important physical consequences?



CP conservation

- To determine if CP is conserved or broken, we study the three weak-basis invariants J_1, J_2, J_3 .
- If J_1, J_2, J_3 are all real, CP is conserved.
- If at least one of J_1, J_2, J_3 is complex, CP is broken.



CP breaking

- If CP is not conserved, we can determine if it is broken spontaneously or explicitly by studying another set of invariants: I_{Y3Z} , I_{2Y2Z} , I_{6Z} , I_{3Y3Z} .
- If CP is not conserved, and at the same time I_{Y3Z} , I_{2Y2Z} , I_{6Z} , I_{3Y3Z} are all zero, then CP is spontaneously broken.
- If CP is not conserved, and at the same time at least one of I_{Y3Z} , I_{2Y2Z} , I_{6Z} , I_{3Y3Z} is non-zero, then CP is broken explicitly.

Assumptions

- Z_2 symmetry only softly broken:

$$\lambda_6 = \lambda_7 = 0$$

- Real "vacuum"

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$\text{with } v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$$

The invariants J_i



$$\operatorname{Im} J_1 = -\frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) \operatorname{Im} \lambda_5$$

$$\begin{aligned} \operatorname{Im} J_2 = & -\frac{v_1^2 v_2^2}{v^8} \left[((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_1^4 + 2(\lambda_1 - \lambda_2) \operatorname{Re} \lambda_5 v_1^2 v_2^2 \right. \\ & \left. - ((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_2^4 \right] \operatorname{Im} \lambda_5 \end{aligned}$$

$$\operatorname{Im} J_3 = \frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) (\lambda_1 + \lambda_2 + 2\lambda_4) \operatorname{Im} \lambda_5$$

The invariants

$$I_{Y3Z}, I_{2Y2Z}, I_{6Z}, I_{3Y3Z}$$

$$I_{Y3Z} = 0,$$

$$I_{2Y2Z} = \frac{1}{4}(\lambda_1 - \lambda_2) \operatorname{Im} [(m_{12}^2)^2 \lambda_5^*]$$

$$I_{3Y3Z} = -\frac{1}{8}(m_{11}^2 - m_{22}^2) [(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2] \\ \times \operatorname{Im} [(m_{12}^2)^2 \lambda_5^*]$$

$$I_{6Z} = 0.$$



CP conservation:

CP is conserved if one or more of the following conditions are satisfied:

- $v_1 = 0$
- $v_2 = 0$
- $\text{Im } \lambda_5 = 0$
- $\lambda_1 = \lambda_2$ and $v_1 = v_2$
- $\lambda_1 = \lambda_2$ and $(\lambda_1 - \lambda_3 - \lambda_4)^2 = |\lambda_5|^2$



CP is broken spontaneously when:

- CP is not conserved
AND in addition one (or both) of the following conditions are satisfied
- $\text{Im} \left[(m_{12}^2)^2 \lambda_5^* \right] = 0$
- $\lambda_1 = \lambda_2$ and $m_{11}^2 = m_{22}^2$

CP is broken explicitly when:

- CP is not conserved or broken spontaneously



Strategy

- Start out with a physical spectrum, mixing angles and "vacuum expectation values". (Pick M_1 , M_2 , M_3 , $\tan \beta = v_2 / v_1$, μ^2 , α_i)
- From this, calculate the parameters of the potential, (λ_i and m_{ij}) and $(M_3)^2$
- Apply theoretical (and experimental) constraints
- Determine the CP properties

Theoretical constraints



- Positivity (potential must be bounded from below)
- Globality (the starting "vacuum" should be the global minimum of the potential)
- Unitarity (unitarity at tree level)

Two cases



Case A

- $M_1 = 100 \text{ GeV}$
 - $M_2 = 300 \text{ GeV}$
 - $M^\pm = 500 \text{ GeV}$
 - $\mu^2 = (200 \text{ GeV})^2$
 - $\tan \beta = 2$
 - $\alpha_1 = -\pi/6$
- Explore the CP properties in α_2 - α_3 space

Case B

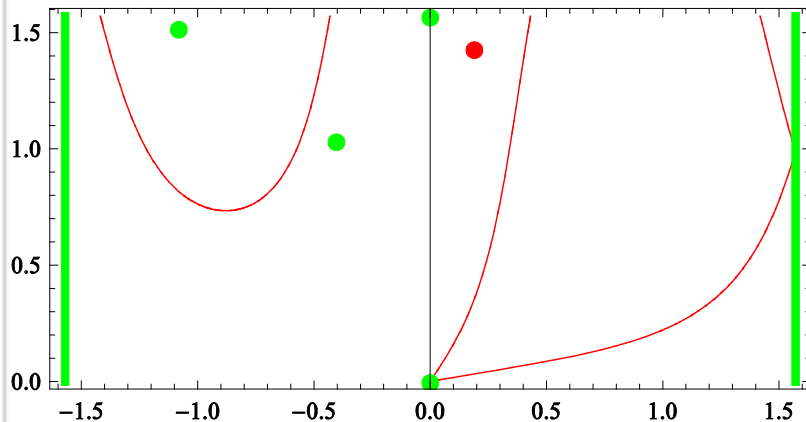
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The CP properties

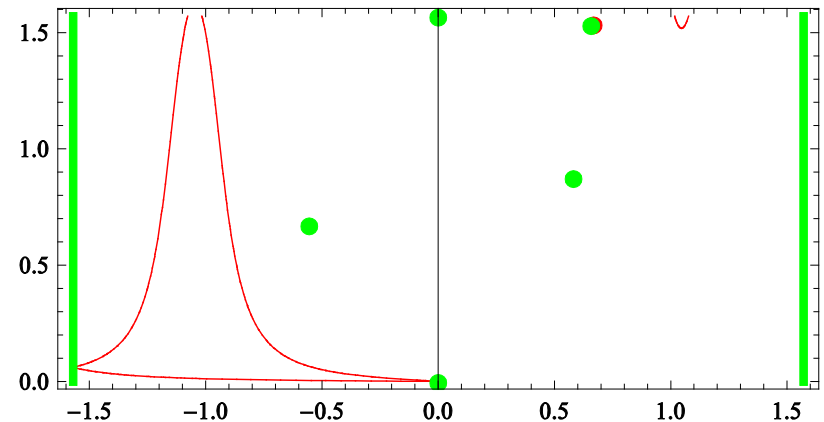


Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$



Green: CPC



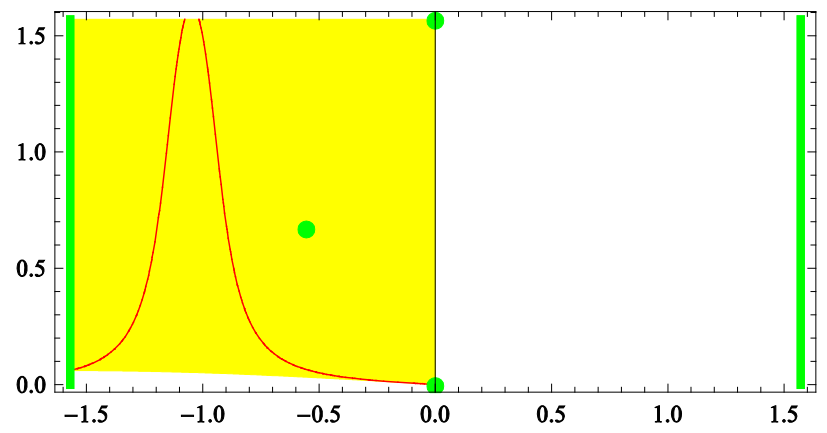
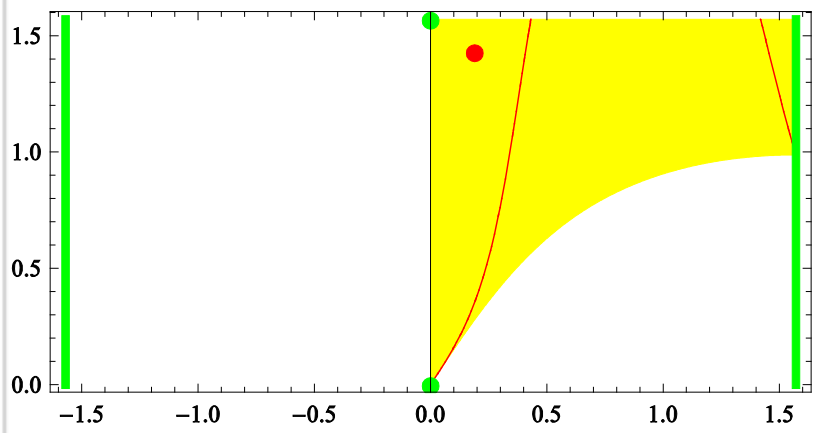
Red: Spontaneous CPV



Region with $(M_3)^2 > (M_2)^2$

Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$

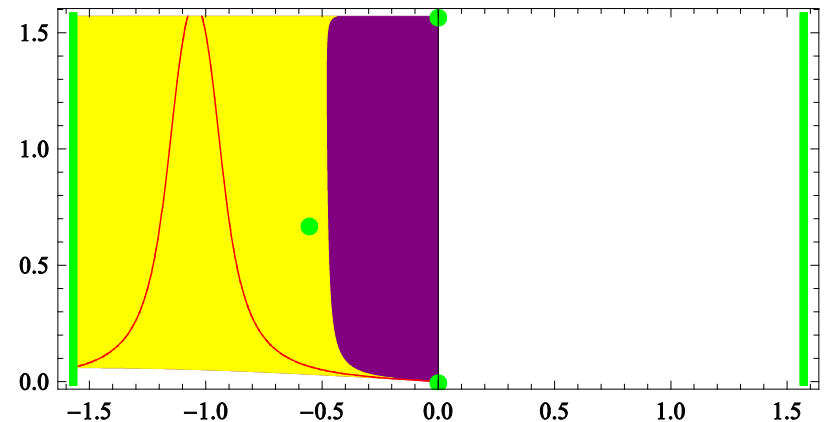
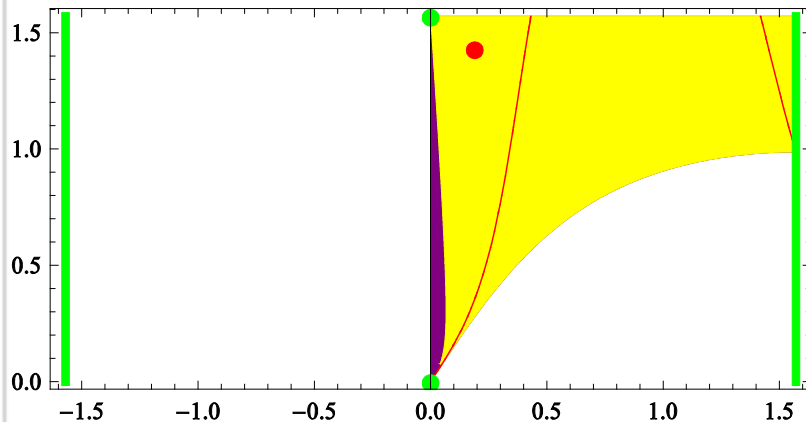


Positivity constraint imposed



Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$

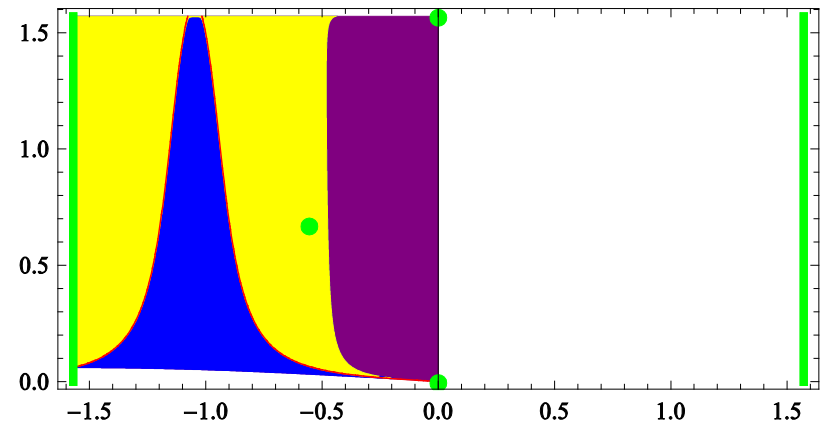
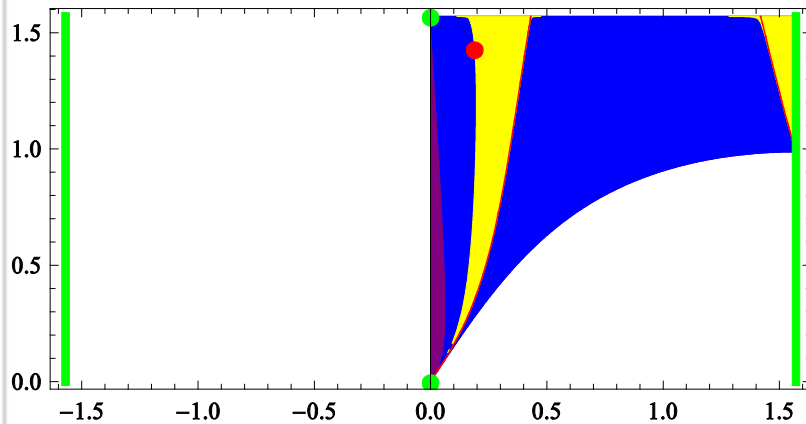


Globality constraint imposed



Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$

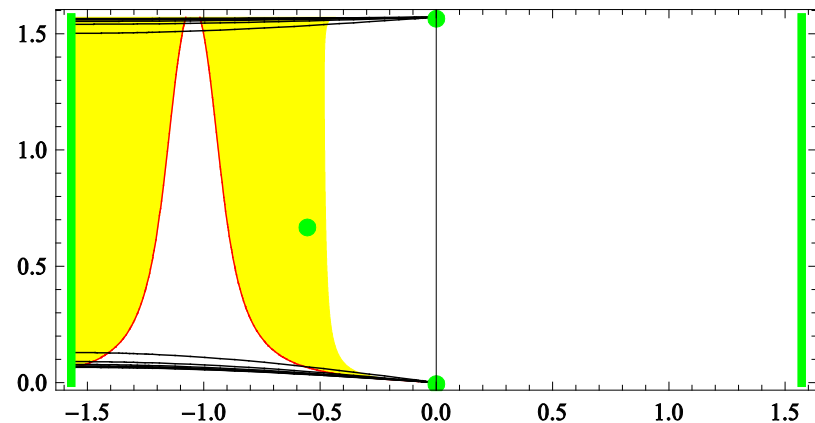
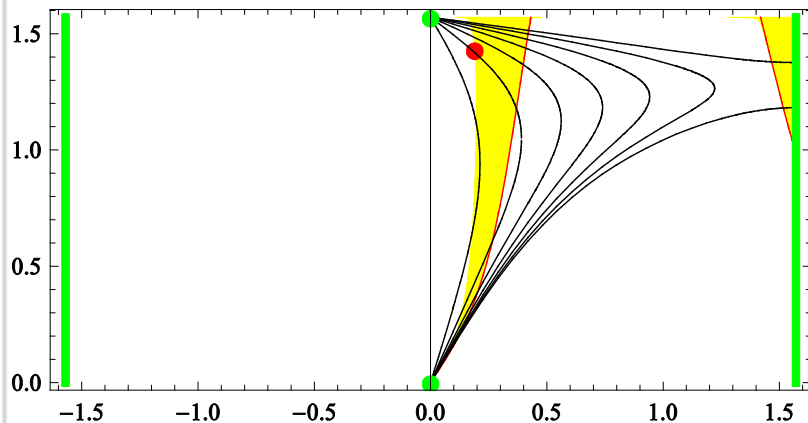




Mass contours for M_3

Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$

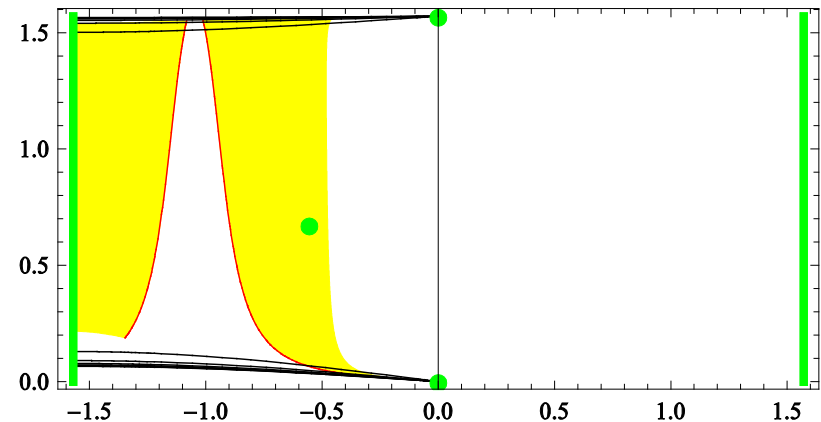
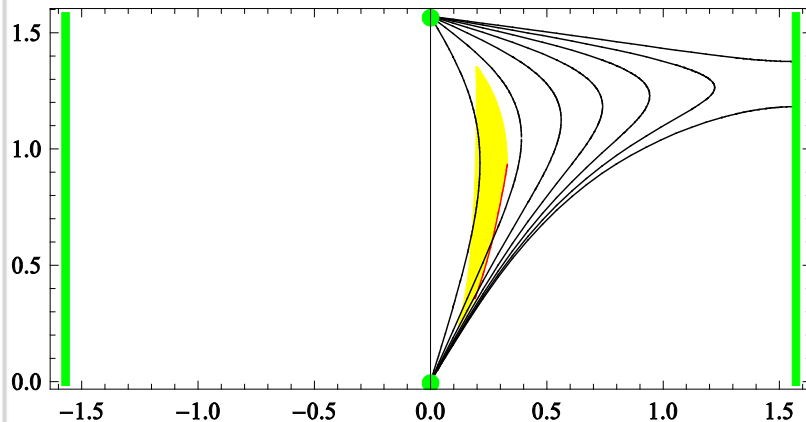


Unitarity constraint imposed



Case A: $\alpha_1 = -\pi/6$

Case B: $\alpha_1 = \pi/6$





Summary

- SCPV only appears along the boundary between the explicitly CP-violating region and the region forbidden because our "vacuum" is not the global minimum.
- When we have SCPV the potential has two minima of exactly the same depth, meaning that the global minimum is not unique.



Open questions

- What are the physical implications of two minima of the same depth?
- Will the universe choose one of these minima, or exist in a mixed state?
- Could the early universe have rested in a "false vacuum" before tunneling into the proper vacuum?
- Is our universe today in a state of a "false vacuum" so that it at a later stage may tunnel into the proper vacuum?